A $U(1)$ squeezing code

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There are several ways of extending squeezing dynamics to rotors with $U(1)$ symmetry. In this short note we will look at one which in some sense is a $U(1)$ generalization of the R squeezing code, defined as follows. Each time an update occurs, the following happens:

1. A site $\mathbf{r}$ is chosen uniformly at random.
2. A direction $\mathbf{d}$ is chosen uniformly at random from the set $\{\pm\hat{\mathbf{x}}, \pm\hat{\mathbf{y}}\}$. Define $\delta\phi = \phi_{\mathbf{r}+\mathbf{d}} - \phi_{\mathbf{r}}$.
3. The spin at site $\mathbf{r}$ is updated according to

$$\phi_{\mathbf{r}} \mapsto \phi_{\mathbf{r}} + \delta\phi \times \begin{cases} \Theta(-\sin(\delta\phi)) & \mathbf{d} \in \{\pm\hat{\mathbf{x}}\} \\ \Theta(\sin(\delta\phi)) & \mathbf{d} \in \{\pm\hat{\mathbf{y}}\} \end{cases} \tag{1}$$

   In words, this means that $\phi_{\mathbf{r}}$ is "infected" to become equal to $\phi_{\mathbf{r}+\mathbf{d}}$ if $\phi_{\mathbf{r}+\mathbf{d}}$ is "behind" $\phi_{\mathbf{r}}$ when $\mathbf{d} \in \{\pm\hat{\mathbf{x}}\}$, or if $\phi_{\mathbf{r}+\mathbf{d}}$ is "ahead of" $\phi_{\mathbf{r}}$ when $\mathbf{d} \in \{\pm\hat{\mathbf{y}}\}$ (where "behind" and "ahead of" refer to relative positions on the unit circle, judged in a counterclockwise-rotating fashion).

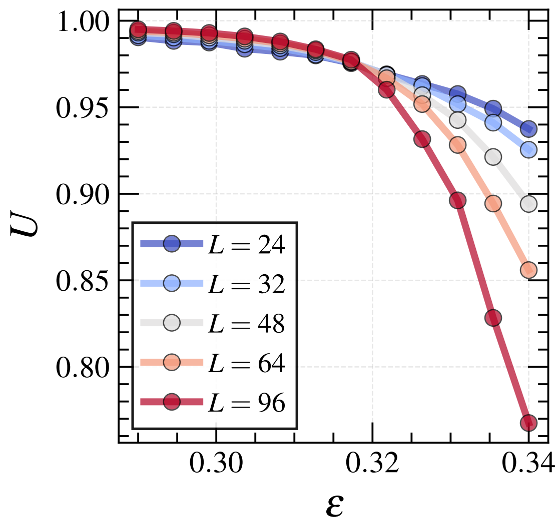
4. Noise occurs on site $\mathbf{r}$ by sending

$$\phi_{\mathbf{r}} \mapsto \phi_{\mathbf{r}} + \varepsilon\xi, \tag{2}$$

   where ξ is uniformly random on the interval $[-1, 1]$.

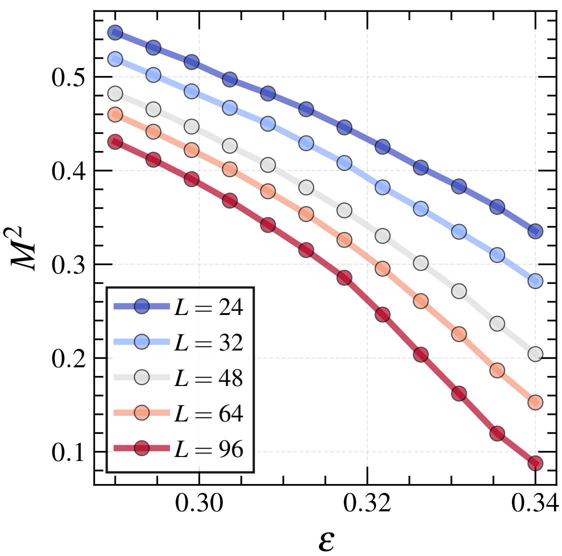
Note that this dynamics has an internal $U(1)$ (viz. a $U(1)$ that is *not* intertwined with spatial rotations), together with a mixed $\mathbb{Z}_2 \circ \mathbb{Z}_2$ symmetry that combines a spin flip $\phi \mapsto -\phi$ with a reflection that exchanges $x \leftrightarrow y$.

The Binder ratio of the magnetization identifies a transition around $\varepsilon \approx 0.315$:



(3)

The fact that the ordered phase only has QLRO is revealed by computing the squared magnetization $\langle M^2 \rangle$:



(4)

which at fixed ε decreases as an inverse power of L. Nevertheless, the dynamics, and the behavior of vortices, is quite different in this model.

Define the relaxation time as

$$t_{\mathsf{rel}} = \mathbb{E} \min\{t \,:\, M(t) > 0.9 M(\infty)\} \tag{5}$$

where $M(t) = L^{-2} |\sum_{\mathbf{r}} \hat{\phi}_{\mathbf{r}}(t)|$ is the magnitude of the magnetization (here $\hat{\phi}_{\mathbf{r}} = (\cos(\phi_{\mathbf{r}}), \sin(\phi_{\mathbf{r}}))$, and $\mathbb{E}$ denotes an average over stochastic evolutions starting from completely random initial states.

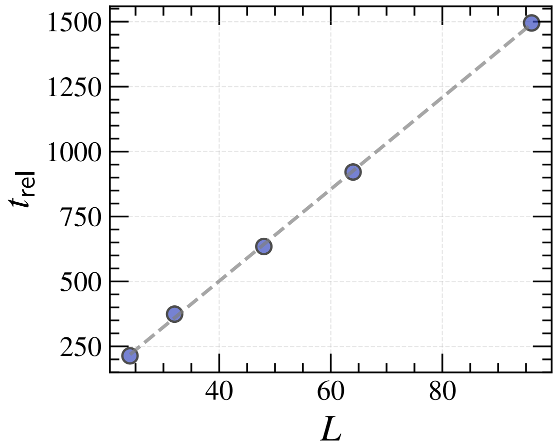
In the XY model, the size of correlation domains following a quench grows as[1]

$$\xi(t) \sim \left(\frac{t}{\ln t}\right)^{1/2}. \tag{6}$$

and t_{rel}, which is set by the time by which $\xi(t_{\mathsf{rel}}) \sim L$, therefore scales as

$$t_{\mathsf{rel}} \sim L^2, \tag{7}$$

up to logarithmic corrections. In the squeezed XY model, the relaxation time is instead ballistic:



(8)

where the dashed line is a fit to $t_{\mathsf{rel}} \sim L$, and the data was computed by averaging over 100 quenches. The squeezing dynamics thus significantly speeds up the relaxation—but this is not enough to get LRO.

[1]The power-law interactions between vortices occur with an attractive force of $1/r$; since $dr/dt \sim -1/r$ gives $r \sim \sqrt{t}$, the interactions do not modify diffusion (apart from the log correction).