Scaling in the RSOS model

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These notes reproduce some of the most basic aspects of the physics of the resticted solid-on-solid model of surface growth, which was "genetically engineered" (phrasing from Zhang's review) by Kim and Kosterlitz to give something that relatively quickly flows to KPZ (in comparison to e.g. Eden or ballistic deposition). Of course none of the conclusions here are new; the purpose of this note is simply to collect a few results to serve as a benchmark against studies of other surface growth models.

Expectations

The microscopic model does random deposition under the "marching soldiers" constraint that $|h(x+1)-h(x)| \leq 1 \; \forall x$. Since valleys and peaks are treated differently the KPZ nonlinearity is present, and since larger values of $|\partial_x h|$ slow down growth (since bigger slopes have more sites that are blocked from updating), we have

$$\partial_t h = \nu \partial_x^2 h + \lambda(\partial_x h)^2 + \eta, \tag{1}$$

where η is the noise and $\lambda < 0$.

The main quantities we will look at will be the second moment of the roughness and the skewness. To this end, define

$$\begin{aligned} \sigma^2(t) &= \frac{1}{L}\int dx \, \langle (h(x,t) - \bar{h}(t))^2 \rangle \\ S_3(t) &= \frac{1}{L\sigma^3(t)}\int dx \, \langle (h(x,t) - \bar{h}(t))^3 \rangle \end{aligned} \tag{2}$$

where $\bar{h}$ is a spatial average and $\langle \cdot \rangle$ a noise average. The germane scaling exponents are

$$\sigma(t) \sim \min(L^\alpha, t^\beta), \tag{3}$$

with the crossover time being at $t \sim L^z, z = \alpha/\beta$. The standard values to compare with are

$$(\alpha, \beta, z) = \begin{cases} (1/2, 1/4, 2) & EW \\ (1/2, 1/3, 3/2) & KPZ \end{cases}. \tag{4}$$

KPZ has $\alpha = 1/2$ since $P(h) \sim e^{-c\int (\partial_x h)^2}$ can easily be checked to be a stationary point of the FP equation, regardless of λ (as the λ term of the deterministic drift part is a total derivative).

The skewness $S_3 = 0$ for EW, since the evolution is just Gaussian and has manifest $h \leftrightarrow -h$ symmetry. For KPZ, S_3 reaches a universal nonzero value in the growing regime of

$$S_3(t) \approx 0.29\,\mathrm{sgn}(\lambda) \qquad (1 \ll t \ll L^z), \tag{5}$$

while $S_3(t \gg L^z) = 0$ on account of the Gaussianity of the late-time steady state. The sign of the skewness matches that of the nonlinearity because e.g. when $\lambda > 0$ the dynamics creates sharper peaks, giving a positively-skewed measure and $S_3 > 0$.

The dynamic exponent z can be obtained from α and β, but a more efficient approach is to measure sample-to-sample fluctuations of the roughness to directly extract the correlation length ξ. In a system of size L, the L/ξ correlation domains will behave as independent random variables as far as the roughness is concerned. Therefore, abusing notation a bit by treating the roughness σ as a sample-dependent random variable, the CLT means

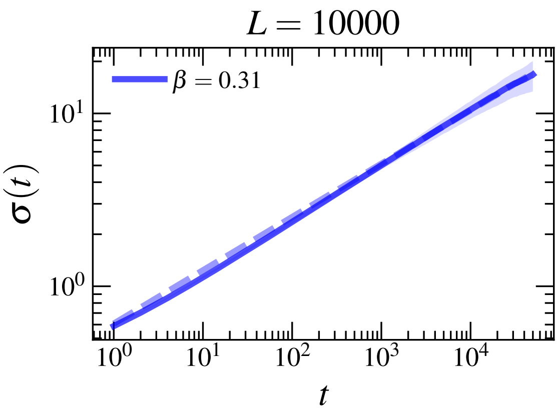
$$\frac{\mathrm{var}(\sigma)}{\langle\sigma\rangle^2} \sim \frac{L}{\xi}, \tag{6}$$

where the $\langle\cdot\rangle$ is a sample average. This provides us with an explicit way to compute ξ, which in the following we will simply define as

$$\xi(t) = L\frac{\langle\sigma(t)\rangle^2}{\mathrm{var}(\sigma(t))}. \tag{7}$$

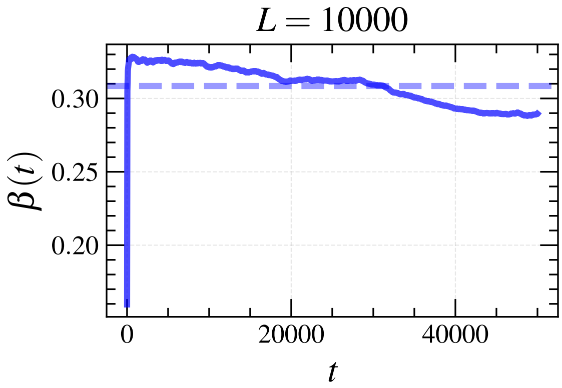
Numerics

Now let us see to what extent the above is borne out in low-budget numerics. Here we set $L = 10^4$ and run 10^3 samples. First, for the roughness (shaded regions drawn at 1σ):



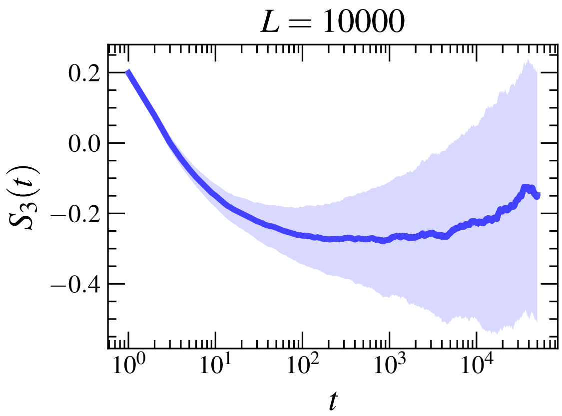
(8)

If we look at the time-dependent exponent $\beta(t)$ on a linear scale, we see that even at this system size and time, there is already a decay towards smaller β:



(9)

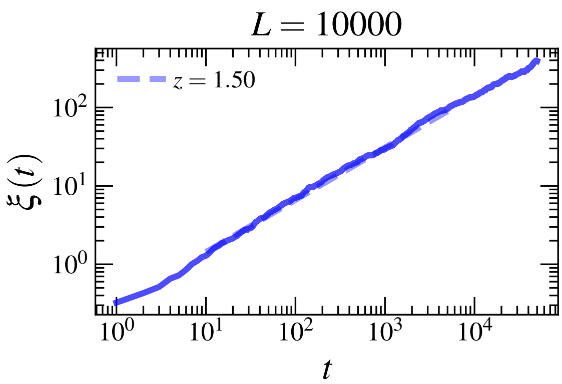
The asymptotic value of the skewness in the growing regime is reached around $t \sim 10^2$, and it starts drifting back towards zero around $t \sim 10^4$:



(10)

where the plateau is at $S_3 \approx -0.28$, close enough to the KPZ expectation.

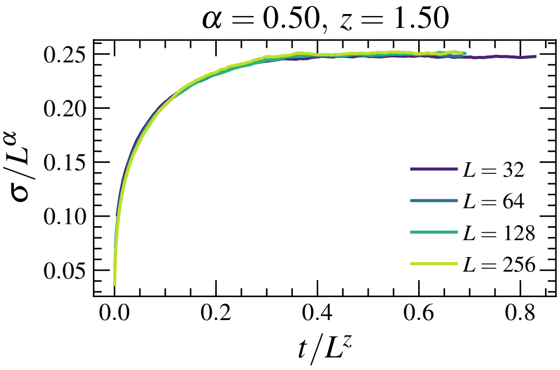
For the correlation length, we find



(11)

where the hard-to-see dashed line is a fit to $z = 3/2$.

To check the crossover to the late-time regime, we plot σ/L^α against t/L^z for different system sizes:



(12)

which gives $\alpha = 1/2$ as expected.